Lyppard Grange – Concrete, Pictorial and Abstract Policy

This policy supports the White Rose Maths scheme used throughout the school. Progression within each area of calculation is in line with the programme of study in the 2014 National Curriculum. This calculation policy should be used to support children to develop a deep understanding of number and calculation, to help to expose the structure and understand the procedures. This policy has been designed to teach children through the use of concrete, pictorial and abstract representations.

- Concrete representation— a pupil is first introduced to an idea or skill by acting it out with real objects. This is a 'hands on' component using real objects and is a foundation for conceptual understanding.
- Pictorial representation – a pupil has sufficiently understood the 'hands on' experiences performed and can now relate them to representations, such as a diagram or picture of the problem.
- Abstract representation—a pupil is now capable of representing problems by using mathematical notation, for example 12 x 2 = 24.

It is important that conceptual understanding, supported by the use of representation, is secure for all procedures. Reinforcement is achieved by going back and forth between these representations. Children should use concrete alongside pictorial and abstract, a CPA approach is NOT progressive and a CPA approach is for ALL children not just for those deemed to need additional support.

Mathematics Mastery

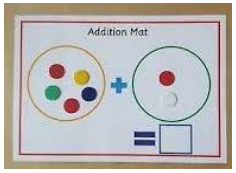
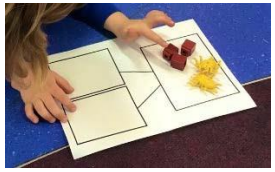
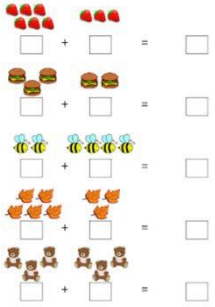
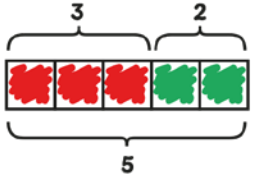
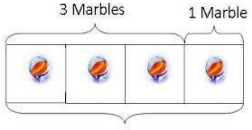
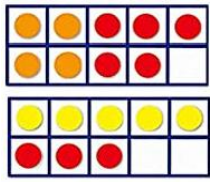
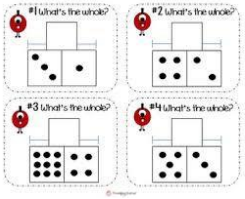
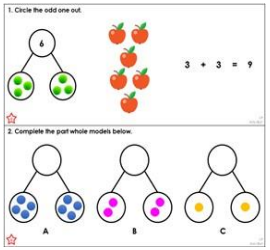
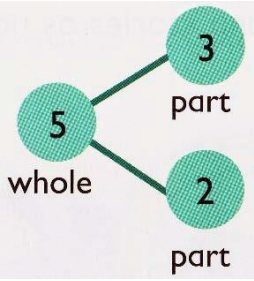
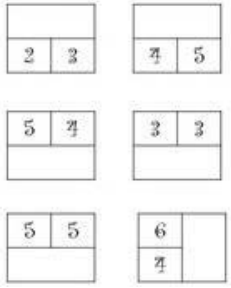
At the centre of the mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This policy outlines the different calculation strategies that should be taught and used in Year 1 to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

How to use the policy:

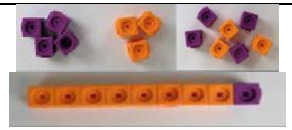
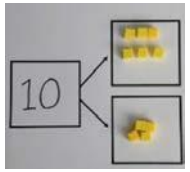
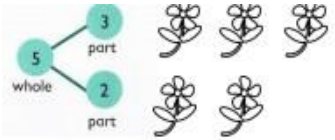
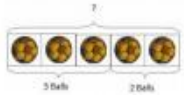
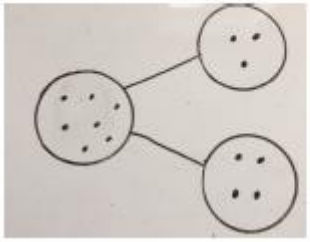
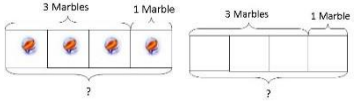
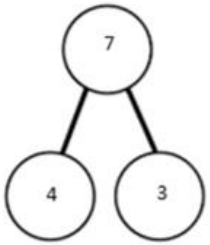
This mathematics policy is a guide for all staff at Lyppard Grange Primary School and has been adapted from a document created by the Stour Academy which in turn was produced using materials from NCTEM. All teachers have been given the scheme of work from the White Rose Maths Hub and are required to base their planning around their year group's modules and not to move onto a higher year group's scheme work. These modules use the Singapore Maths Methods and are affiliated to the workings of the 2014 Maths Programme of Study. Teachers can use any teaching resources that they wish to use and the policy does not recommend one set of resources over another, rather that, a variety of resources are used.
For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract (CPA) approach [Make it, Draw it, Write it] is for children to have a true understanding of a mathematical concept, they need to master all three phases within a year group's scheme of work.

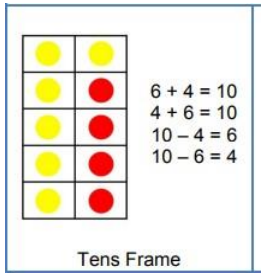
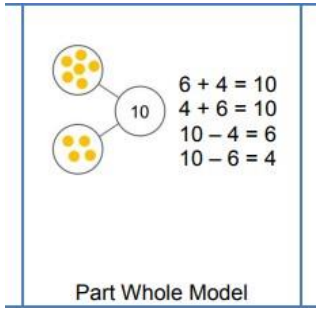
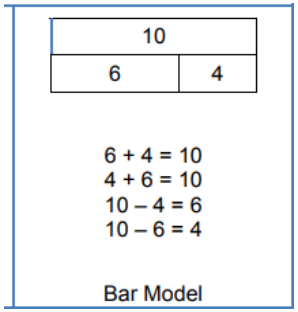
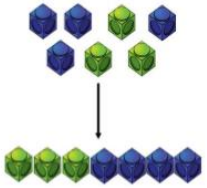
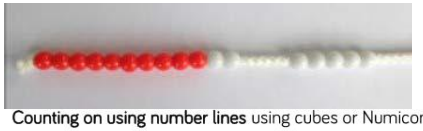
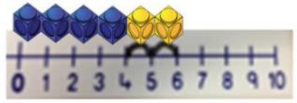
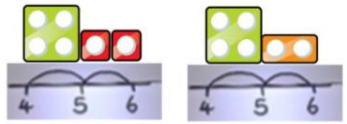
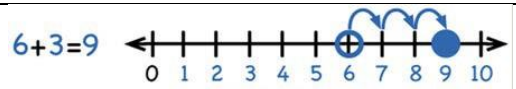
Addition

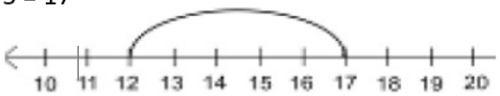
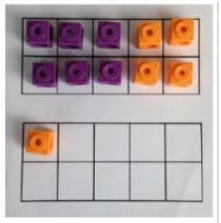
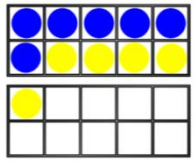
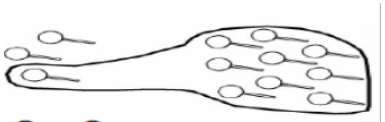
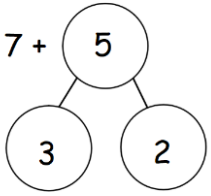
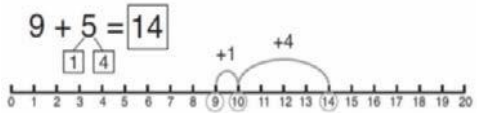
Addition- EYFS

	Concrete	Pictorial	Abstract
- Have a deep understanding of number to 10, including the composition of each number; -Subitise (recognise quantities without counting) up to 5; -Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts. - Verbally count beyond 20, recognising the pattern of the counting system; - Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity;	   Use toys and general classroom resources for children to physically manipulate, group/regroup.  Use specific maths resources such as counters, multilink cubes, Numicon etc.    Use visual supports such as ten frames, part, part, whole and addition mats, with the physical objects and resources that can be manipulated.	  Two groups of pictures so children are able to count the total.  Bar model using visuals, pictures/icons or colours.   Use visual supports such as ten frames, part, part, whole and addition mats with pictures/icons.  	A focus on symbols and numbers to form a calculation. $5+2=7$   * No expectation for children to be able to record a number sentence/addition calculation.

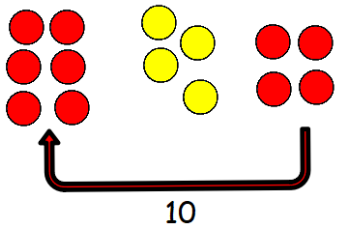
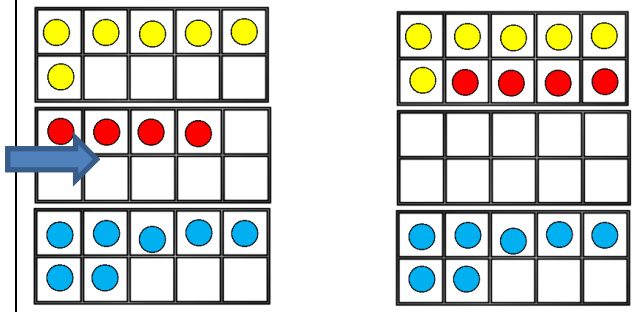
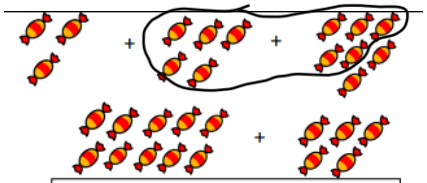
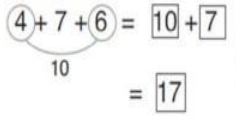
Addition- Year 1

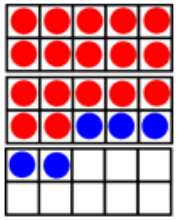
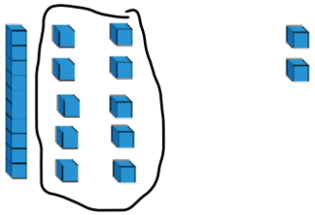
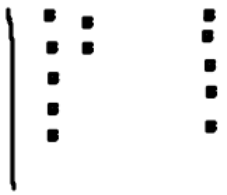
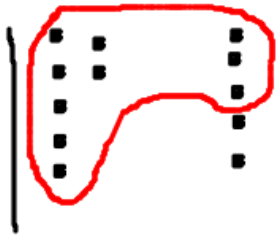
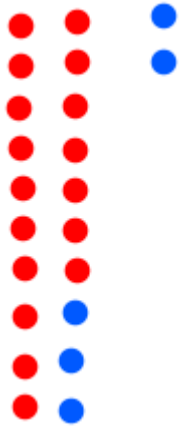
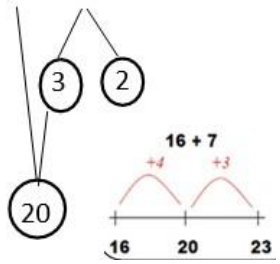
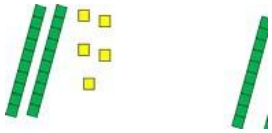
Objective and Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	 Use cubes and counters/double sided counters to add two numbers together as a group or in a bar. (Some children may still need to use real objects)  Use part-part whole model 	 Use pictures to add two numbers together as a group or in a bar.   Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.  The Bar Model will be continued from EYFS as a method to support problem solving involving addition, continuing with the concrete representations and moving onto using pictorial representations of objects. Some children will also move onto the abstract.  Pictorial (concrete) Abstract	 Use the part-part whole diagram as shown above to move into the abstract. **4 + 3 = 7** **10= 6 + 4** **4 + 3 = 7** **Four is a part, 3 is a part and the whole is seven.** 

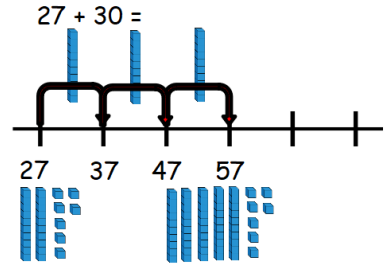
Represent and use number bonds and related subtraction facts within 20	 (Some children may need to initially use real objects then move onto the representation, double sided counters grab ten – throw them down what calculations can you see? Egg boxes, cake tins, patterned wrapping paper with 10 spaces may also be used to support this)		 Bar model and part-part whole to be used alongside abstract
Addition and subtraction of one-digit and two-digit numbers to 20 including 0. Counting on using number lines using cubes or Numicon.	Count all  Count on   Counting on using number lines using cubes or Numicon.  	 Start at the larger number on the number line and count on in ones.	5 + 12 = 17 17= 12 + 5 It is important that calculations are represented in a variety of ways (e.g. with the perceived answer at the start) – to emphasise meaning of =.

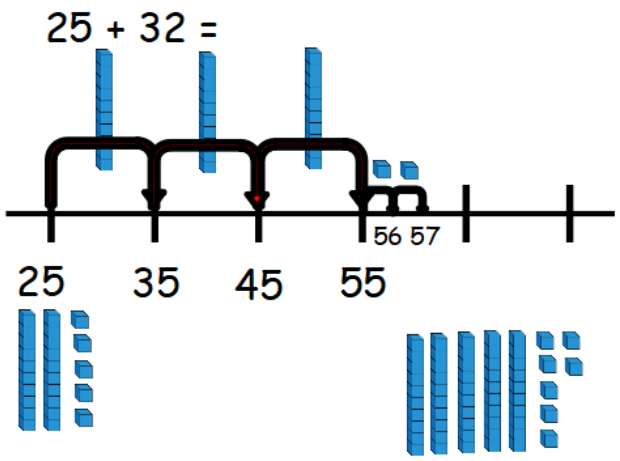
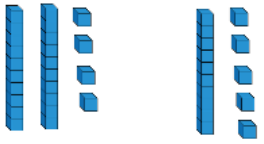
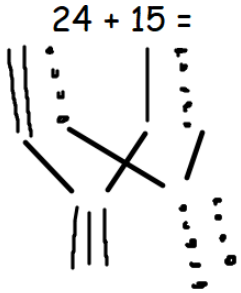
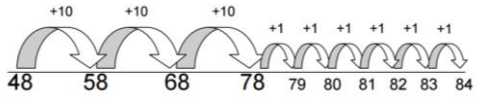
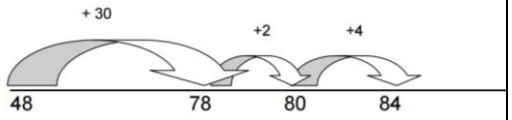
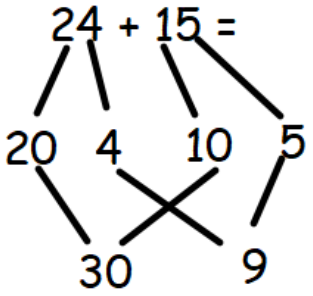
Start at the bigger number and counting on	Start with the larger number on the bead string and then count off to the smaller number 1 by 1 to find the answer. See above for use of other materials.	8 + 7 = 15  12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10 (The 'Make 10' strategy)	Regrouping to make 10; using ten frames and counters/cubes or using Numicon. 6 + 5   6 + 5 = 11  Start with the bigger number and use the smaller number to make 10. Use ten frames.	Children to draw the ten frame and counters/cubes.   3 + 9 = Use pictures or a number line. Regroup or Partition the smaller number using the part, part whole model to make 10.  	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I to add on now? 
Vocabulary	addition, add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, balancing, part, part, whole, bar model, ones		

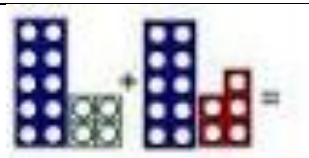
Addition- Year 2

Objective and Strategy	Concrete	Pictorial	Abstract
Adding 3 1-digit numbers	**4 + 7 + 6= 17** Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit. Use double-sided counters.  	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.

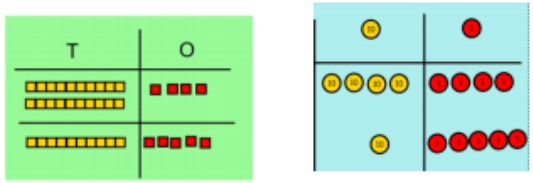
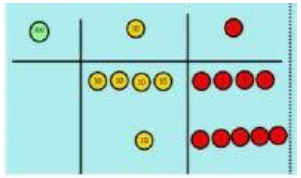
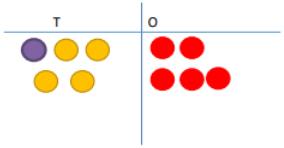
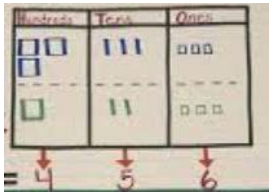
Adding a 2-digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32  Use base ten to make 'magic ten' and add on the rest.  10 Use bead strings and Numicon in the same way as the Year 1 - Regrouping to make 10 (The 'Make 10' strategy)	 Use sticks and dot to represent tens and ones – then circle to make 'magic ten, then add on the rest.  Draw out dots, add on what is needed to make 'magic ten'. 	17 + 5 = 22 Use part part whole and number line to model. 
Adding a 2-digit number and multiples of 10	 25 + 10 = 35 Explore that the ones digit does not change	Use Sticks and dots to represent the base ten. 23 +10 = 33 + 10 = 43 + 10 =  The calculation will be shown	

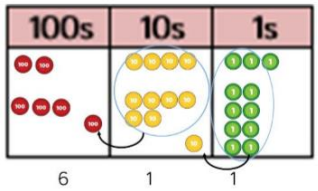
	This could also be explored using the bead strings, if you build the ones from the right hand side and slide the tens in from the left.	alongside the representations to help cement the connections.	Base ten or sticks and dots, could be used on the number line initially to link the concrete pictorial and abstract. Explore patterns such as: - 27 + 10 = 37 27 + 20 = 47 27 + □ = 57 

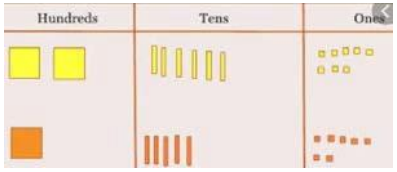
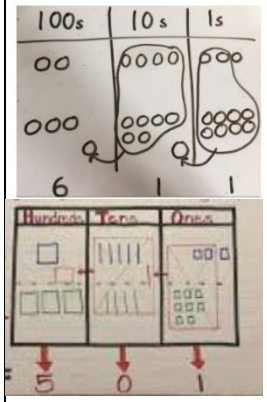
Adding two 2-digit numbers (No re-grouping)	<u>Number lines</u>  <u>Partitioning</u> Base ten or Numicon should be used first to ensure children have secure understanding of place value and the value of each part of the number (some children may be ready for place value counters but this is not essential at this stage) Partition the numbers into tens and ones 24 + 15 =  Add together the ones first and then add the tens. (Some children may not be ready for place value counters in Y2) Numicon may also be used	<u>Number lines</u> Draw out the number line using stick and dot to represent the tens and ones. <u>Partitioning</u>  	<u>Number lines</u>  Children continue to use blank number lines in order to add 2 digit numbers and beyond. E.g. 48+36=  Children will add in 1 chunk of 30 before adding a chunk of 2 single units to bridge to 10 then a chunk of 4 single units. <u>Partitioning:</u>  4 + 5 = 9 20 + 10 = 30 30 + 9 = 39

			
Vocabulary	add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary		

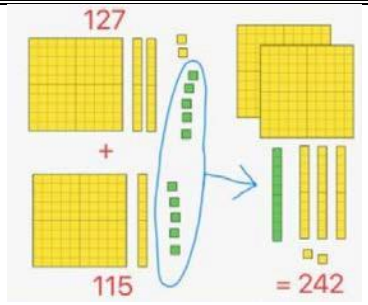
Addition- Year 3

Objective and Strategy	Concrete	Pictorial	Abstract
Add and subtract numbers with up to 3-digits, using formal written methods of columnar addition Column addition (no regrouping)	**24 + 15=** Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  Using manipulatives (dienes, Numicon, counters), children are to line up hundreds, tens and ones.  44 + 15 = 40 + 4 10 + 5 50 + 9 Children should be secure with using PV counters before moving onto pictorial. The calculation will be shown alongside the model used to see the connection **Model** | **Calculation**	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.   Children are to draw, in a PV frame, the manipulatives, that they are using. Secure knowledge of representation with the PV columns. The calculation will be shown alongside the model to see the connection **Model** | **Calculation**	Children to move onto recording more formally. 40 + 7 30 + 5 70 + 12

<table>
<tr>
<td>Column addition (with regrouping)</td>
<td> 


</td>
<td> 

Children can draw a representation of the grid to further support their understanding, carrying the ten ***underneath*** the line.</td>
<td>564 + 137 =
500 + 60 + 4
+ 100 + 30 + 7
700 + 00 + 1
100 10

Children are to begin with the abstract: expanded form.</td>
</tr>
<tr>
<td></td>
<td> 

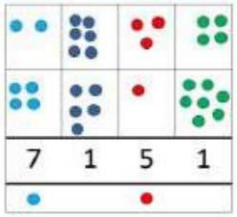
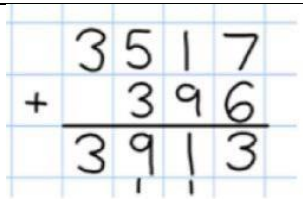
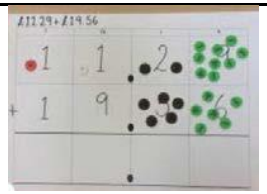
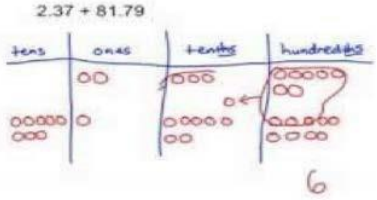
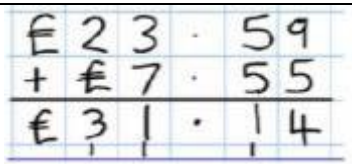
Exchange ten ones for a ten. Model using Dienes, Numicon and place value counters.

The calculation will be shown alongside the model used to see the connection

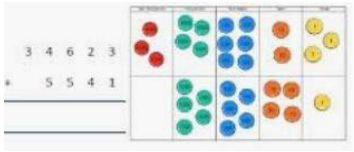
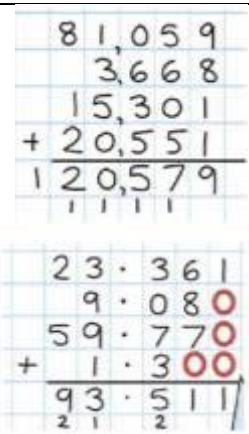
<table>
<tr><th>Model</th><th>Calculation</th></tr>
<tr><td></td><td></td></tr>
</table></td>
<td>The calculation will be shown alongside the model used to see the connection

<table>
<tr><th>Model</th><th>Calculation</th></tr>
<tr><td></td><td></td></tr>
</table></td>
<td></td>
</tr>
<tr>
<td>Vocabulary</td>
<td colspan="3">addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, partition, whole part, hundreds, tens, ones</td>
</tr>
</table>

Addition- Year 4

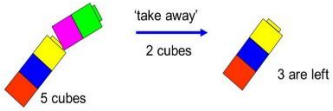
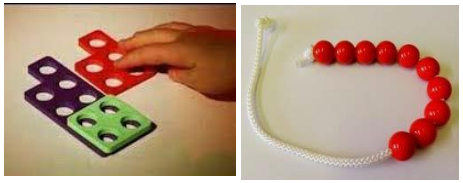
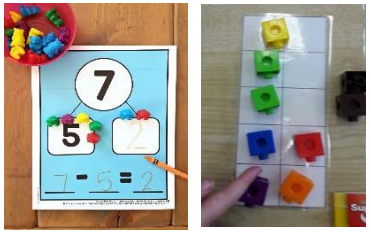
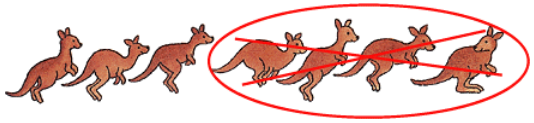
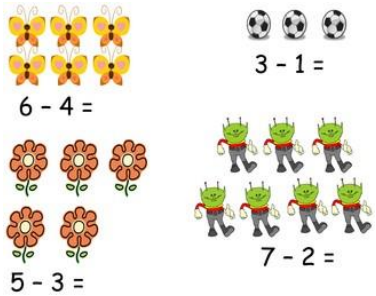
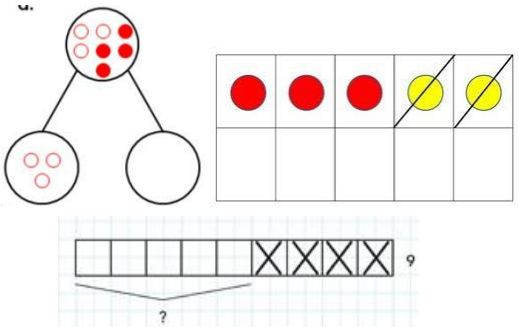
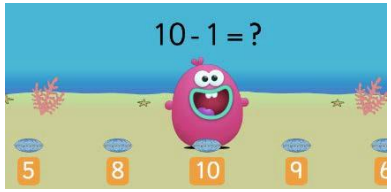
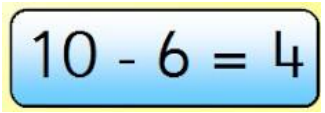
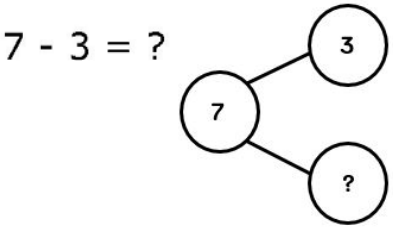
Objective and Strategy	Concrete	Pictorial	Abstract
Using formal written methods of columnar addition where appropriate add numbers with up to 4 digits (with exchange)	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.  The calculation will be shown alongside the model used to see the connection. **Model** | **Calculation**	 Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding. The calculation will be shown alongside the model used to see the connection. **Model** | **Calculation**	 Continue from previous work to carry hundreds as well as tens.
Add decimals with 2 decimal places, including money.	 Introduce decimal place value counters and model exchange for addition.		 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.
Vocabulary	addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, decimal, decimal point		

Addition- Year 5/6

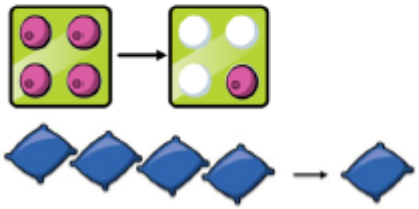
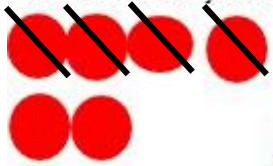
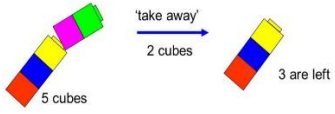
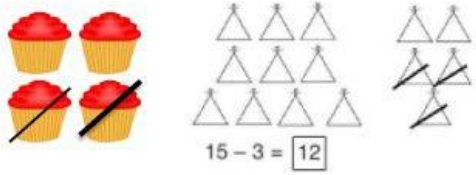
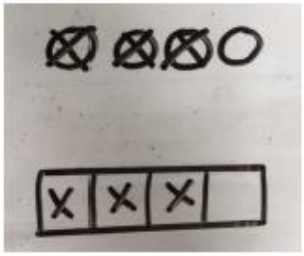
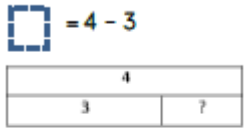
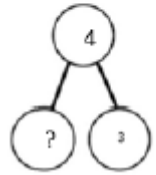
Objective and Strategy	Concrete	Pictorial	Abstract
add numbers with more than 4 digits.	See Year 4  The calculation will be shown alongside the model used to see the connection. **Model** | **Calculation**	See Year 4 The calculation will be shown alongside the model used to see the connection. **Model** | **Calculation**	Children should have abstract supported by a pictorial or concrete initially until method is secure.
add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points.	See Year 4 and extend to work with numbers of increasing value.	See Year 4	 Insert zeros for place holders.
Vocabulary	addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, decimal, decimal point		

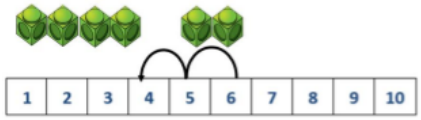
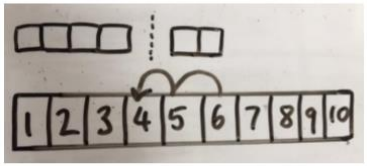
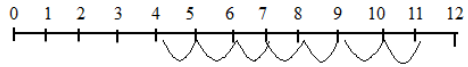
Subtraction

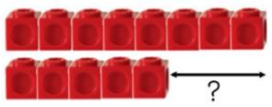
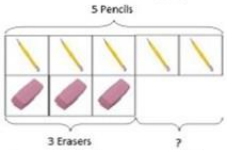
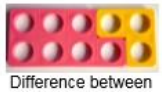
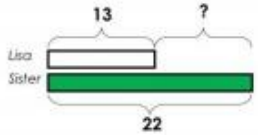
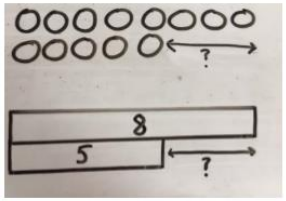
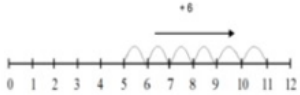
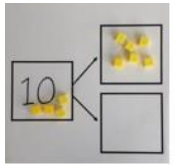
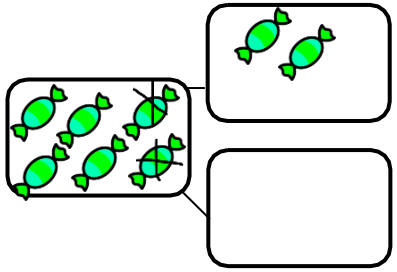
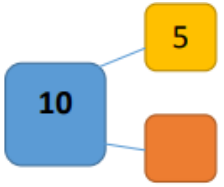
Subtraction- EYFS

Objectives	Concrete	Pictorial	Abstract
- Have a deep understanding of number to 10, including the composition of each number; -Subitise (recognise quantities without counting) up to 5; -Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts. - Verbally count beyond 20, recognising the pattern of the counting system; - Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity;.	 Use toys and general classroom resources for children to physically manipulate, group/regroup.   Use specific maths resources such as multi-link cubes, Numicon, bead strings etc.  Use visual supports such as ten frames, part, part, whole and subtraction mats, with the physical objects and resources that can be manipulated.	  A group of pictures for children to cross out or cover quantities to support subtraction.  Use visual supports such as ten frames, part, part, whole and bar model with pictures/icons.	A focus on symbols and numbers to form a calculation.     * No expectation for children to be able to record a number sentence/addition calculation.

Subtraction- Year 1

Objective and Strategy	Concrete	Pictorial	Abstract
Subtract one-digit and two-digit numbers to 20, including 0. Taking away ones	Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 – 3 = 1  Use physical objects, counters, cubes etc to show how objects can be taken away.  6—4 = 2  4—2 = 2   Use Cuisenaire rods to explore subtraction and support the concept of barmodelling  	Cross out drawn objects to show what has been taken away.  Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.   	7—4 = 3 16—9 = 7 4- 3 =  

Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. **13 – 4**  Use counters and move them away from the group as you take ther away counting backwards as you go.  **Counting back** (using number lines or number tracks) children start with 6 and count back 2. 6 - 2 = 4  	Children to represent what they see pictorially e.g.  Count back on a number line or track, draw on the jumps as you subtract each one. Start at the bigger number and count back the smaller number showing the jumps on the number line. **At Lyppard Grange we count backwards under the line so that the children can see the difference between addition where we count on the top and subtraction.**	Put 13 in your head, count back 4. What number are you at? (Use your fingers to help you) 11 – 7  Children record drawing jumps on prepared lines.

Find the difference	**Finding the difference** (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5.  Compare objects and amounts  7 'Seven is 3 more than four' 4 'I am 2 years older than my sister'  Lay objects to represent bar model. Difference between 13 and 8 13 – 8 = _ 8 + _ = 13  Number bonds  (Ten frame)  Difference between 7 and 10	**Comparison Bar Models** Draw bars to find the difference between 2 numbers. *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.*  Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate. 	Find the difference between 8 and 5. 8 – 5, the difference is ☐ Children to explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference. Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister?  Count on to find the difference.
Represent and use number bonds and related subtraction facts within 20 Part-part whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what s the other part? 10—6 = 4	 Use a pictorial representation of objects to show the part-part whole model	10 – 5 = 10 - ___ = 5  Move to using numbers within the part whole model..

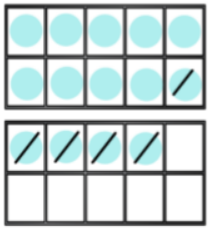
<table>
<tr>
<td>Make 10</td>
<td>

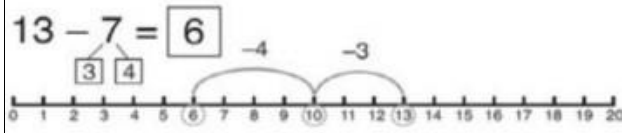
14 – 9 =

Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.

</td>
<td>

Children to present the ten frame pictorially and discuss what they did to make 10.

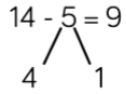




Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.

</td>
<td>

Children to show how they can make 10 by partitioning the subtrahend.



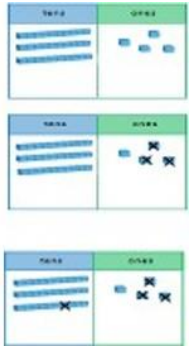
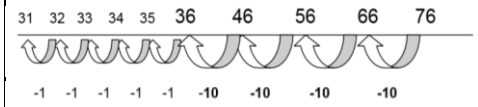
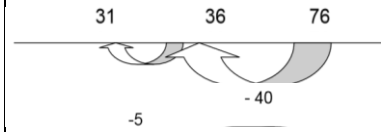
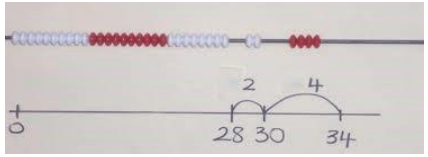
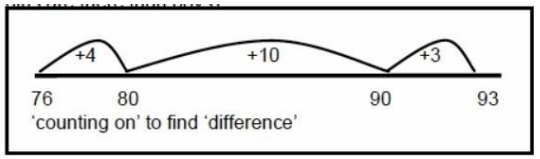
14 – 4 = 10
10 – 1 = 9

16 – 8=

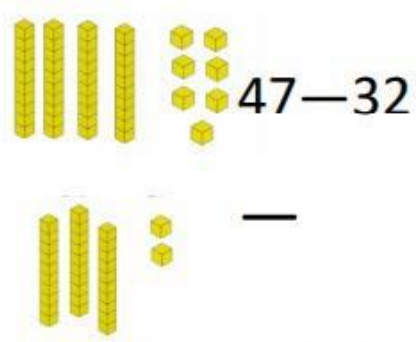
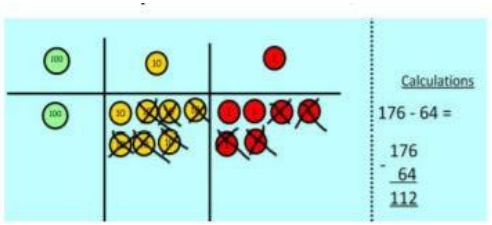
How many do we take off to reach the next 10?
How many do we have left to take off?

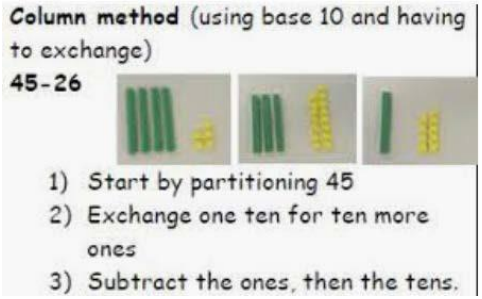
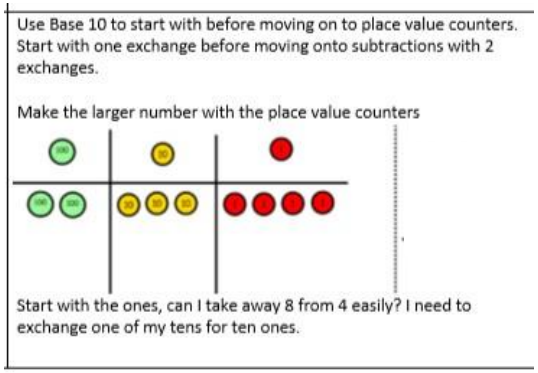
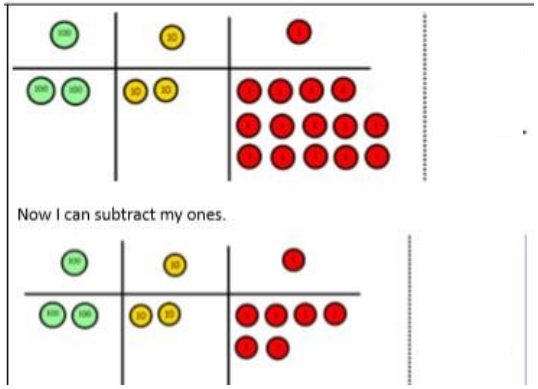
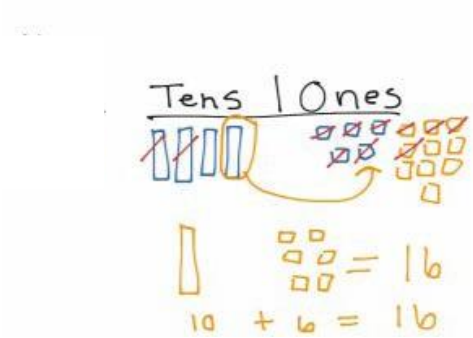
</td>
</tr>
<tr>
<td>Vocabulary</td>
<td colspan="3">equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is…, part part whole, find the difference, partition.</td>
</tr>
</table>

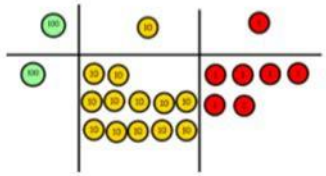
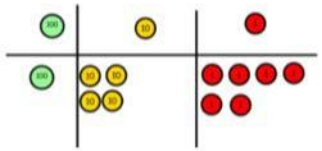
Subtraction- Year 2

Objective and Strategy	Concrete	Pictorial	Abstract
Subtract a two-digit number and ones, a two-digit number and tens, two two-digit numbers Partitioning to subtract without re-Grouping: *'Friendly numbers'*	Use bead strings/rekenreks as a number line – create the larger number subtract tens and then the ones. 34—13 = 21  Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.  43—21 = 22	Children to use blank number lines in order to subtract 2 digit numbers and beyond. E.g. 76-45=  Children will subtract 4 lots of 10 before subtracting 5 single units. Children continue to use blank number lines in order to subtract 2 digit numbers and beyond by using more efficient jumps. E.g. 76-45= 
Make ten strategy – useful when crossing the ten. Or when finding the difference between two numbers. Counting on to find the difference	**Finding the difference.**  34—28 Use a rekenrek or bead strings to model counting to next ten and the rest.	**Finding the difference.**  Use a number line to count on to next ten and then the rest.	93—76 = 17
Vocabulary	equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…difference, count on, strategy, partition, tens, ones, part part whole, find the difference.		

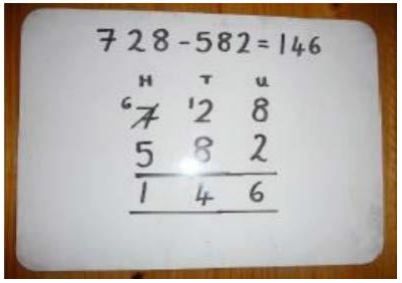
Subtraction- Year 3

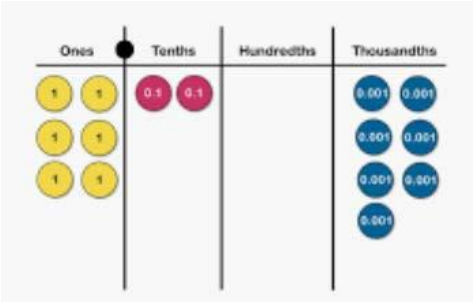
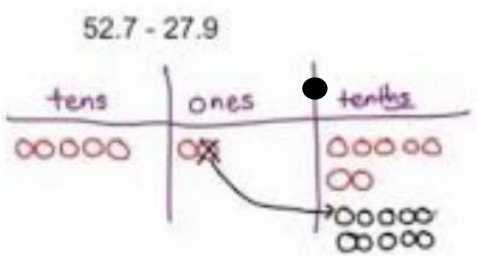
Objective and Strategy	Concrete	Pictorial	Abstract
To subtract numbers with up to three-digits, using formal written methods of columnar subtraction Column subtraction (without exchanging)	 Use base 10 or Numicon to model Remove the base ten as it is subtracted. The calculation will be shown alongside the model chosen to see the connection **Model** | **Calculation**	Children are to be secure with use of PV counters before moving onto abstract.  Children could use squares sticks and dots to represent Base 10. 	Children should begin with the expanded form.  *Recording subtraction in columns supports place value and prepares for formal written methods with larger numbers.* e.g. 43-21=22 40 + 3 -20 + 1 20 + 2 476 – 123 = 353 400 + 70 + 6 - 100 + 20 + 3 300 + 50 + 3

Column Subtraction (with exchanging)	Begin with base 10 or Numicon. Move to PV counters, modelling the exchange of a ten into ten ones. Use the term 'exchange'.    	 When confident, children can find their own way to record the exchange/regrouping Again, use the pictorial image alongside the formal method to support understanding.	Children should begin with the expanded form. Moving onto a more formal way as below (bottom picture). 

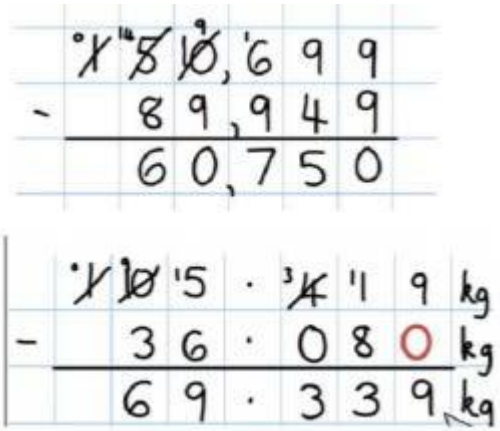
	Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Calculations 234 - 88 Now I can take away eight tens and complete my subtraction  Calculations ~~2~~ 1 ~~3~~ 4 - 88 146 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		
Vocabulary	equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…difference, count on, strategy, partition, tens, ones, hundreds, exchange		

Subtraction- Year 4

Objective and Strategy	Concrete	Pictorial	Abstract
Subtract numbers with up to 4 digits using the formal written methods appropriate of columnar subtraction where appropriate Year 4 subtraction with up to 4 digits.	Model process of exchange using Numicon, base ten and then move to PV counters. Use the term 'exchange' see Year 3. The calculation will be shown alongside the model chosen to see the connection **Model** | **Calculation**	Children to draw PV counters and or base ten, show their exchange—see Y3 The calculation will be shown alongside the model chosen to see the connection **Model** | **Calculation**	 This will lead to an understanding of subtracting any number including decimals.

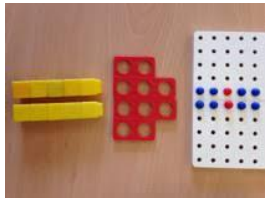
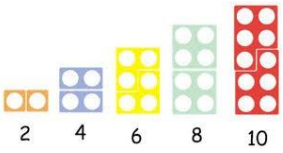
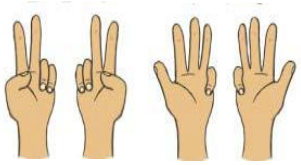
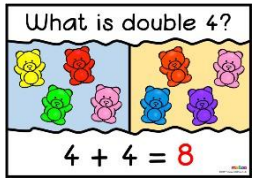
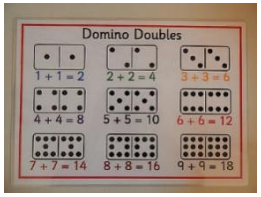
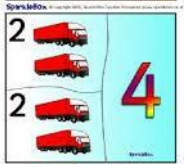
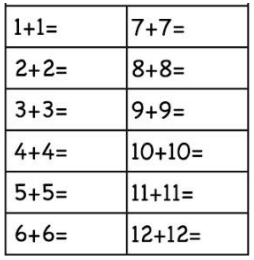
Introduce decimal subtraction through context of money	Children to be encouraged to use counters to represent numbers and take counters away to subtract. 	 When confident, children can find their own way to record the exchange/regrouping	45.7 – 34.8 = 4 45.7 - 34.8 10.9 6 85.70 - 43.05 42.65
Vocabulary	equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…difference, count on, strategy, partition, thousands, hundreds, tens, ones, exchange, decimal, tenths, hundredths		

Subtraction- Year 5/6

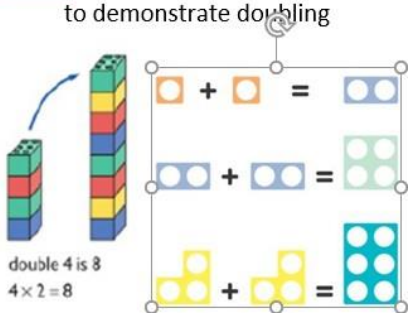
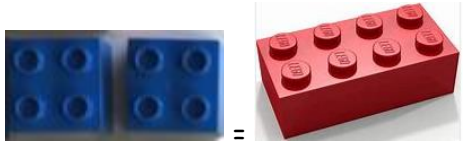
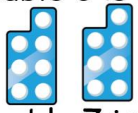
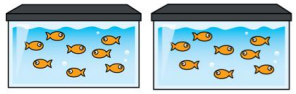
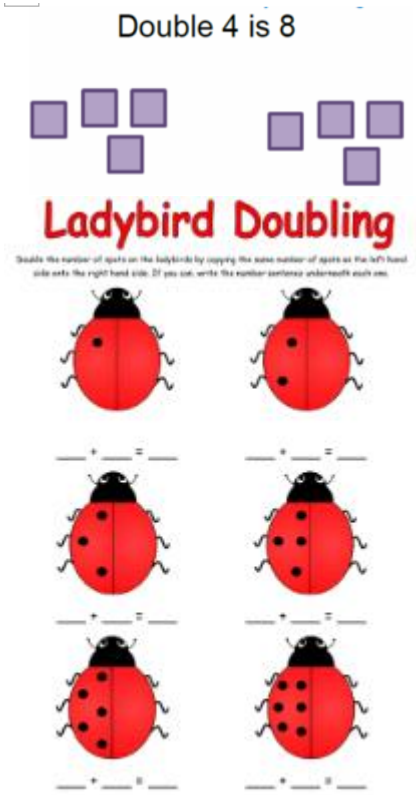
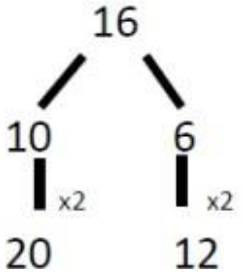
Objective and Strategy	Concrete	Pictorial	Abstract
Subtract with at least 4 digits, including money and measures. Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).	See Year 3 and 4	See Year 3 and 4	
Vocabulary	equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…difference, count on, strategy, partition, thousands, hundreds, tens, ones, exchange, decimal, tenths, hundredths		

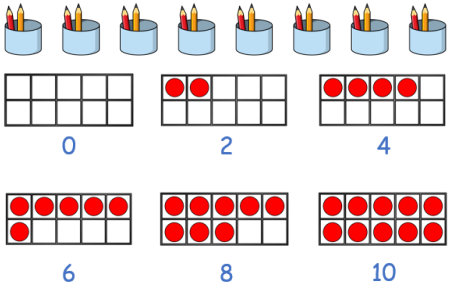
Multiplication

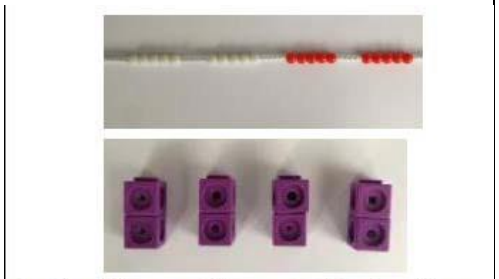
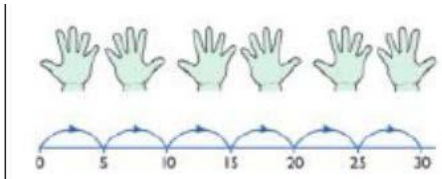
Multiplication-EYFS

Objectives	Concrete	Pictorial	Abstract
- Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.	   Counting and other maths resources for children to make 2 equal groups.    Physical and real life examples that encourage children to see concept of doubling as adding two equal groups. Use Numicon and mirrors to show the number and then double it.	    Pictures and icons that encourage children to see concept of doubling as adding two equal groups.	 Addition calculations to model adding two equal groups.

Multiplication- Year 1

Objective and Strategy	Concrete	Pictorial	Abstract
Doubling	Use Numicon and mirrors to show the number and then double it. Use practical activities using manip-ultives including cubes and Numicon to demonstrate doubling  Use Lego   Double 6 is 12  Double 7 is 14  Double 8 is	Draw pictures to show how to double numbers  See EYFS ideas in addition.	<table><tr><td>1+1=</td><td>7+7=</td></tr><tr><td>2+2=</td><td>8+8=</td></tr><tr><td>3+3=</td><td>9+9=</td></tr><tr><td>4+4=</td><td>10+10=</td></tr><tr><td>5+5=</td><td>11+11=</td></tr><tr><td>6+6=</td><td>12+12=</td></tr></table> Addition calculations to model adding two equal groups.  Partition a number and then double each part before recombining it back together.

<table>
<tr>
<td>Counting in multiples</td>
<td>Emphasis on Equal groups. Use real objects – count out first group and then build others ‘the same’. Counting them loud on multiple and whisper when not e.g. (whisper 1) Loud 2! Etc
 

 
Count in multiples supported by concrete objects in equal groups.</td>
<td> 

Use a number line or pictures to continue support in counting in multiples.

Use clear counters as you count to support, the counting. Or add colour.

<table>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr>
<tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr>
<tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr>
<tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr>
</table></td>
<td>Count in multiples of a number aloud.

Write sequences with multiples of numbers.

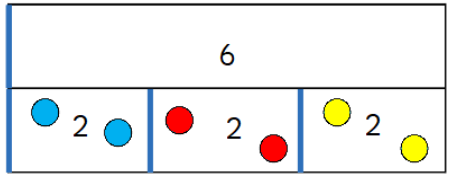
2, 4, 6, 8, 10

5, 10, 15, 20, 25, 30</td>
</tr>
<tr>
<td>Repeated addition</td>
<td>Use real life objects such as socks, gloves etc.
Use cuisinere, Numicon, counters etc to build equal groups.
Then add them together.

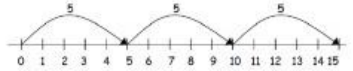
 
Use different objects to add equal groups.</td>
<td>Write addition sentences to describe objects and pictures.

 
2 + 2 + 2 + 2 + 2 = 10

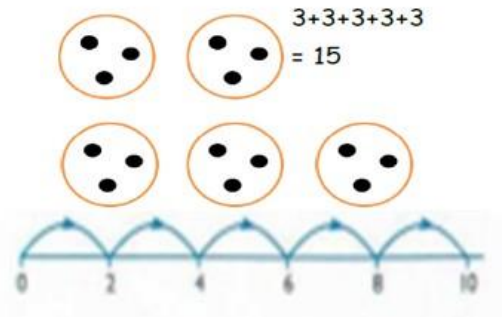
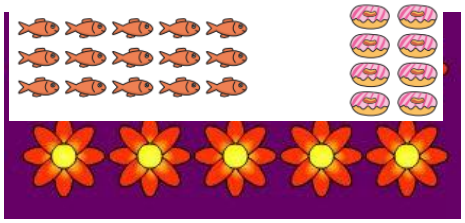
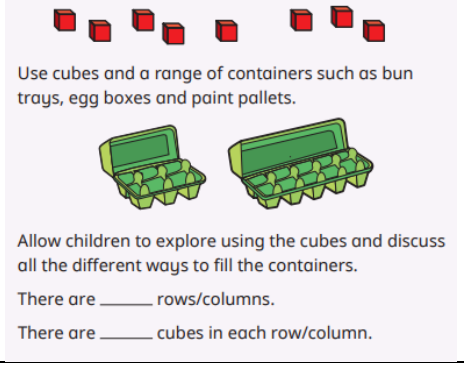
Create bar models to represent the calculations

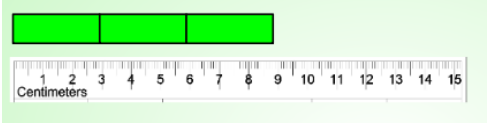
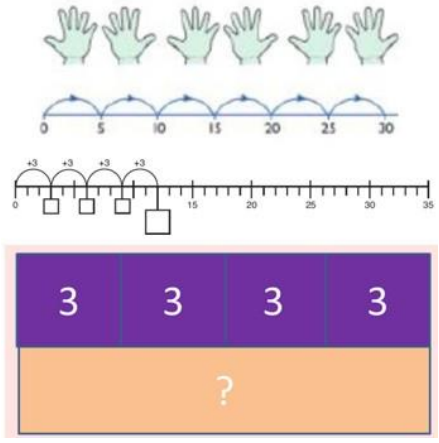
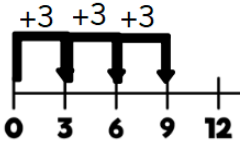
Place counters on the bar model to support calculation.</td>
<td> 

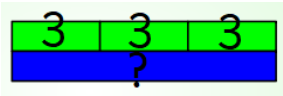
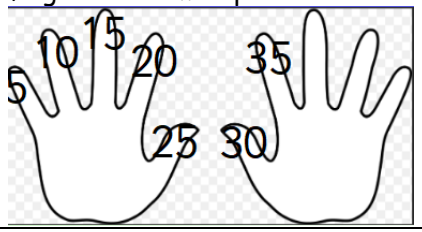
2 + 2 + 2 = 6
 

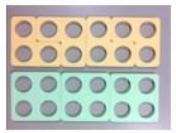
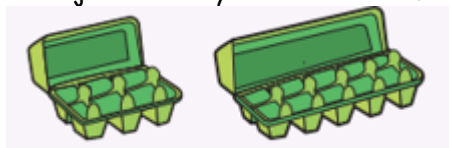
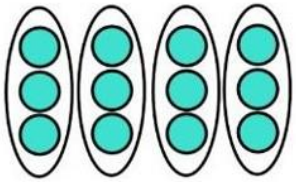
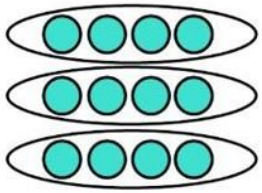
5 + 5 + 5 = 15</td>
</tr>
</table>

		Use pictorial including number lines to solve problems There are 3 sweets in one bag. How many sweets are in 5 bags altogether?  	
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.   	 Draw and paint arrays for different numbers, describe them using rows and columns and altogether.	3 x 2 = 6 Three rows of two. Three lots of 2. 2 x 5 = 10 2 groups of 5. 2 lots of 5.
Vocabulary	Groups of, lots of, times, array, altogether, multiply, rows, columns		

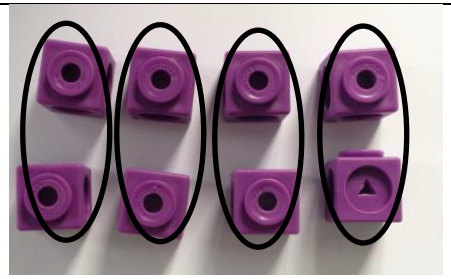
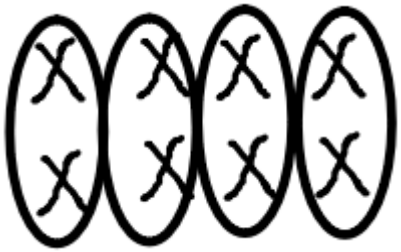
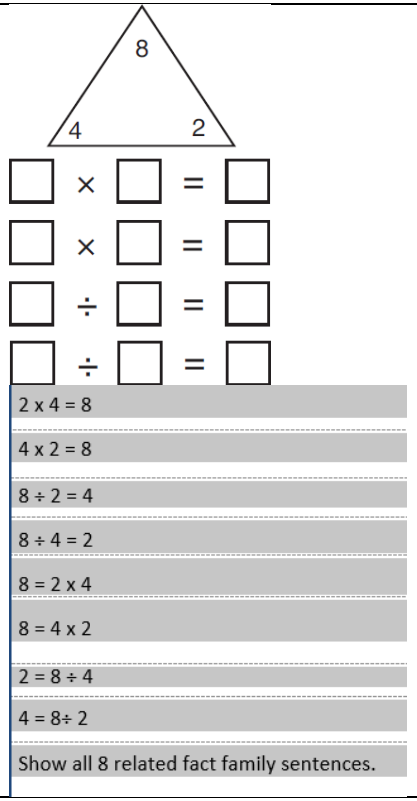
Multiplication- Year 2

Objective and Strategy	Concrete	Pictorial	Abstract
Counting in multiples of 2, 3, 5, 10 from 0 (repeated addition)	Count groups of items using skip counting. Group items into equal groups and use skip counting to find out how many altogether. Use bead strings, numicon, cuisinere and Rekonreks. Place objects into bar models, label the contents, to support transition into pictorial.  3 + 3+ 3 5+ 5+ 5+ 5+ 5   Use cuisinere rods – use 2 pieces when counting in 2s and 3 pieces when counting in 3s. Have a number line/ruler to line up the three's pieces to find out the total. Count as laying them on. Try with different numbers.  	Number lines, counting sticks and bar models should be used to show repre-sentation of counting in multiples.  Could use paper chains - label each part of the chain and then build it to show skip counting. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30  3 + 3 + 3 + 3 = 12 Sing multiples songs – e.g. 3 times table song uptown funk mr demaio 5 times table cowboy song.

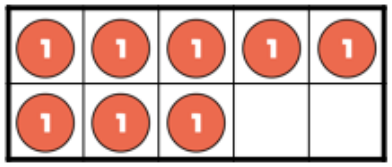
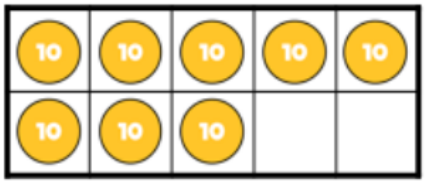
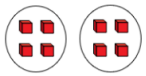
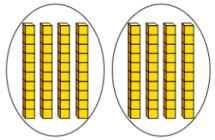
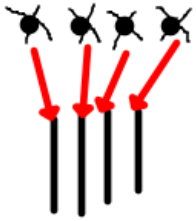
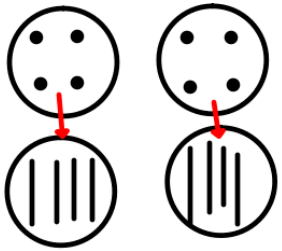
	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40  Use cuisinere on a bar model. So 3 add another 3 add another 3.  Step count on fingers where each finger is the multiple – 		

Multiplication is commutative *Not covered in WR small steps s needs building in.*	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.   Describe the arrays without moving them but circling columns and rows – use the sentence stem There are ____ groups of ___ there are ____ altogether. Use egg boxes or bun trays, or other fixed array patterns build it one way then physically turn it around so children can see it doesn't change the total just the way it is described. 	Use representations of arrays to show different calculations and explore commutativity.  	12 = 3 × 4 12 = 4 × 3 Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

<table>
<tr>
<td>Using the Inverse

This should be taught alongside the commutative law and division, so pupils learn how they work alongside each other.</td>
<td> 
Create an array using cubes/counters etc to show the multiplication. – 4 x 2 or 4 groups of 2. Describe it using: There are ___ groups of ___ there are ___ altogether.
Then look at the altogether and group(divide) it into groups of 2 how many groups of 2 are there?
How are the two representations the same?</td>
<td>Draw arrays
 
Talk about the pictorial representation in the same way as the concrete.</td>
<td> 
</td>
</tr>
<tr>
<td>Vocabulary</td>
<td colspan="3">Groups o
f, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative.</td>
</tr>
</table>

Multiplication- Year 3

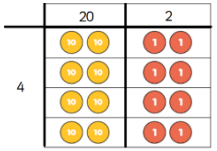
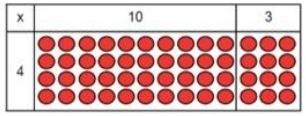
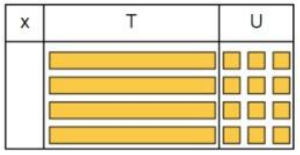
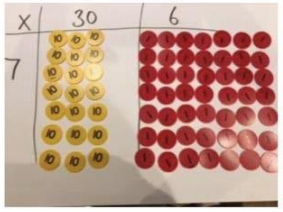
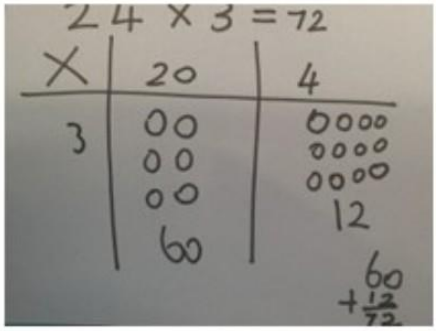
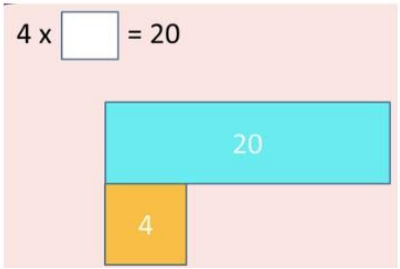
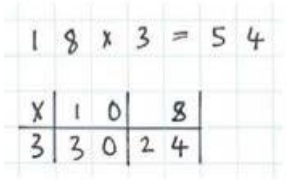
Objective and Strategy	Concrete	Pictorial	Abstract
Working with multiples of 10. Using related calculations. Solving problems including scaling problems. Also a Year 4 objective	 What number is shown. Make it ten times bigger by replacing each 1 with a 10.  Use counters or base 10 to explore related facts in multiplication and division. E.g.  2 × 4 ones = 8 ones 2 × 4 = 8 So…  2 × 4 tens = 8 tens 2 × 40 = 80	Draw ones and then make each one a ten. On the white board rub out or cross out each 1 as you turn it into the ten.   2 x 4 ones = 8 ones So 2 x 40 = 80	If 2 x 4 = 8 What does 2 x 40 =? What other facts do we also know? 4 x 2 = 8 so 40 x 2 = ? 20 x 4 = ? so 4 x 20 = ?

$16 \div 2 = 8$ $16 \div 4 = 4$

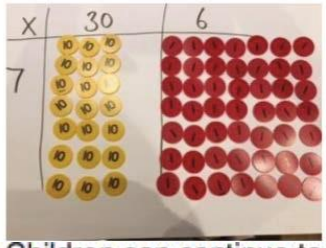
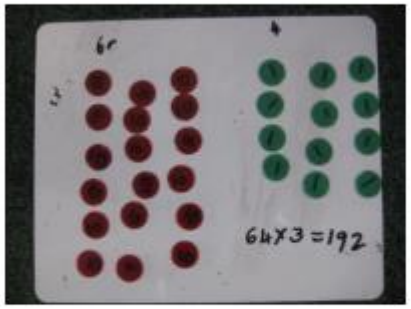
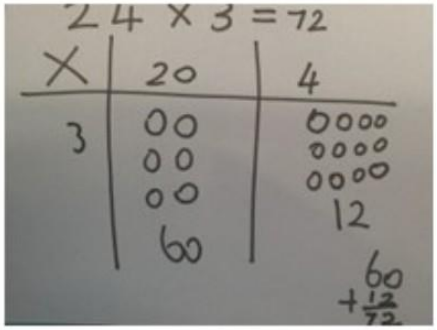
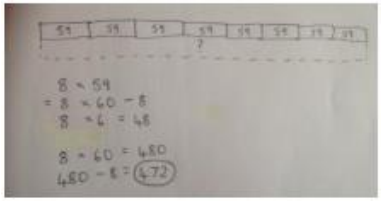
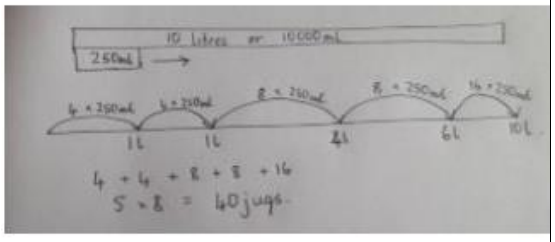
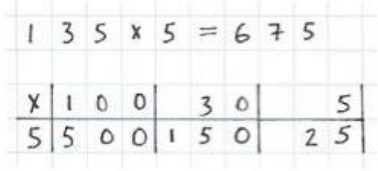
So...

$160 \div 2 = 80$ $160 \div 4 = 40$

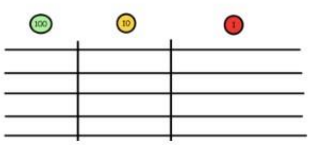
Multiplication- Year 3

Objective and Strategy	Concrete	Pictorial	Abstract
Multiplying two digit number by a one digit number Grid method Solving problems including missing number problems,	 Use place value counters to build the calculation in a grid– e.g. build 22 four times to represent 22 x 4. Could also use base 10 for children who need this level of support. Where needed could lay out the whole number in 1s.  Move on to using Base 10 to move towards a more compact method.   Add up each column, starting with the ones making any exchanges needed. The calculation will be shown alongside the model chosen to see the connection. **Model** | **Calculation**	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  Bar model are used to explore missing numbers 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. TO x O  Children to add up each column to find the answer.
Vocabulary	Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, multiples		

Multiplication- Year 4

Objective and Strategy	Concrete	Pictorial	Abstract
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout Grid method recap from year 3 for 2 digits x 1 digit Multiplying numbers by 1 digit (year 4 expectation)	 Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  Bar modelling and number lines can support learn when solving problems with multiplication alongsi formal written methods.   	HTO x O  Children to add up each column to find the answer.

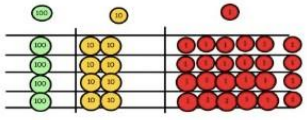
Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.



Calculations

4 x 126

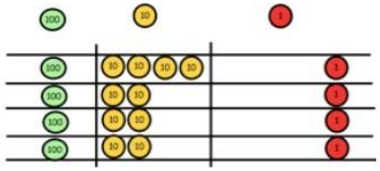
Fill each row with 126.



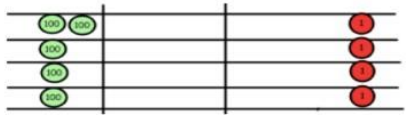
Calculations

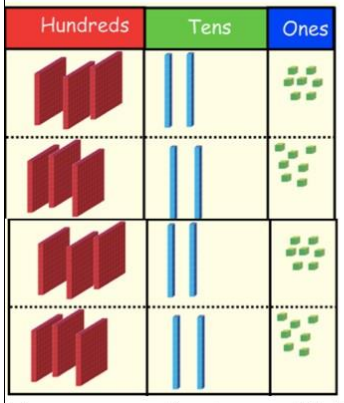
4 x 126

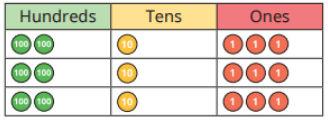
Add up each column, starting with the ones making any exchanges needed.

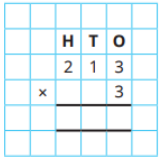


Then you have your answer.

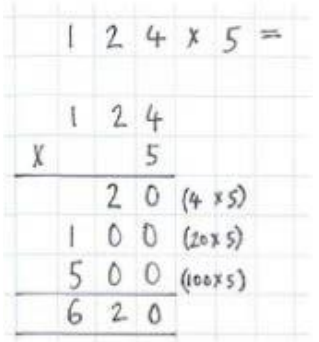


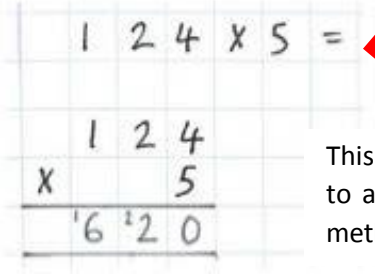
<table>
<tr><td></td><td></td><td></td><td></td></tr>
<tr>
<td>Column multiplication
Expanded column multiplication leading to short multiplication.</td>
<td>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642
 

It is important at this stage that they always multiply the ones first.
The calculation will be shown alongside the model chosen to see the connection
 


</td>
<td>This grid method may be used to show how this relates to a formal written method.

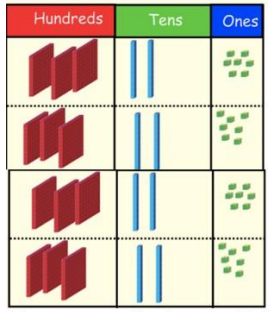
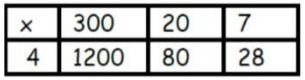
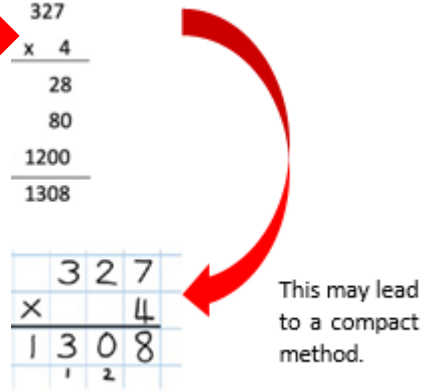
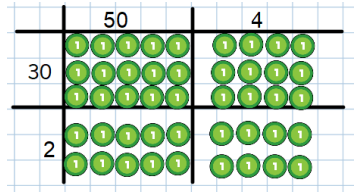
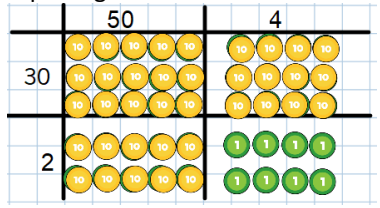
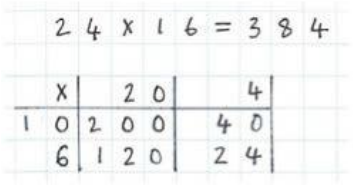
×	100	20	4
5	500	100	20

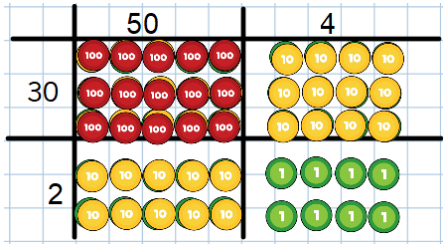
Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.
 
</td>
<td> 



This may lead to a compact method.</td>
</tr>
<tr>
<td>Vocabulary</td>
<td colspan="3">Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive</td>
</tr>
</table>

Multiplication Year 5

Objective and Strategy	Concrete	Pictorial	Abstract
Multiply numbers up to 4-digits by a one-digit number using the format written method, including long multiplication for 2-digit numbers Column multiplication for 3 and 4 digits x 1 digit	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside See strategies for Year 3 and 4 for ideas.	 See Year 3 and 4 for ideas.	
Column multiplication - multiplying 2 digit numbers x 2 digit numbers and 2 digit x 3 digit and 2 digit by 4 digit.	Manipulatives may still be used with the corresponding long multiplication modelled alongside.  Create the number in ones Then multiply by 10 where required by replacing each one with a 10.  Repeat where needed to multiply by 10 again (so that you have multiplied by 100).	Moving forward, multiply by a 2 digit number showing the different rows within the grid method.  The calculation will be shown alongside the model chosen to see the connection. **Model** | **Calculation**	**Expanded long multiplication (1)** 23 x 13 = 299 23 X 13 9 (3 x 3) 60 (3 x 20) + 30 (10 x 3) 200 (10 x 20) 299 This then leads to: **Compact long multiplication (1)** 23 x 13 = 299 23 X 13 + 69 (3 x 23) 230 (10 x 23) 299

<table>
<tr>
<td></td>
<td>


Then add them altogether making any exchanges needed.

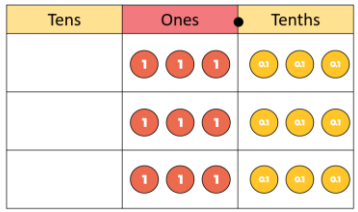
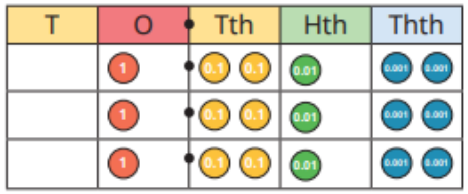
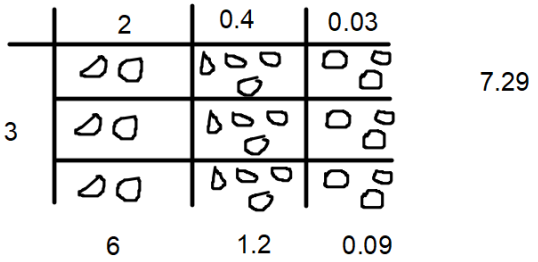
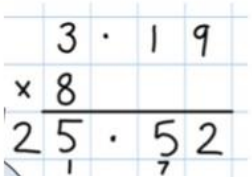
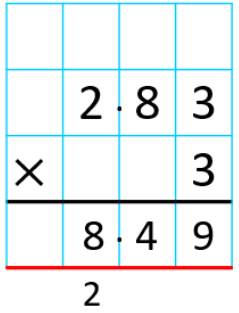
The calculation will be shown alongside the model chosen to see the connection.
<table>
<tr><th>Model</th><th>Calculation</th></tr>
<tr><td></td><td></td></tr>
</table>
</td>
<td></td>
<td>Once children are secure in this method, they then are extended to larger 2 digit calculations and 2 digit x 3 digit calculations.

124 x 26 = 3224

<table>
<tr><td></td><td>1 2 4</td><td></td></tr>
<tr><td>X</td><td>2 6</td><td></td></tr>
<tr><td></td><td>$7^{1}4^{2}4$</td><td>(6x124)</td></tr>
<tr><td>+</td><td>2 4 8 0</td><td>(20x124)</td></tr>
<tr><td></td><td>3 2 2 4</td><td></td></tr>
<tr><td></td><td>1 1</td><td></td></tr>
</table>

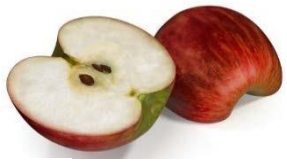
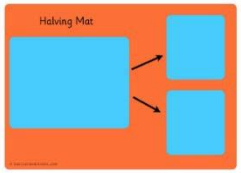
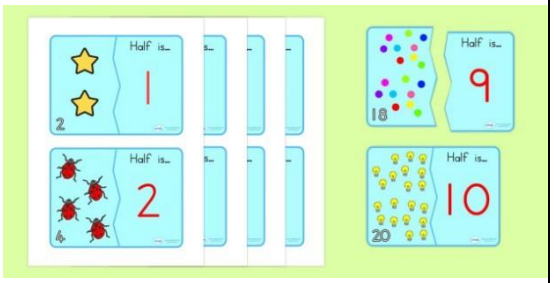
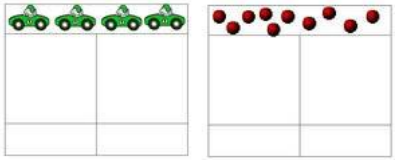
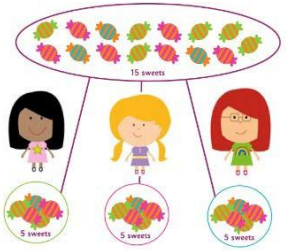
The prompts in brackets can be omitted if children no longer need them.</td>
</tr>
<tr>
<td>Vocabulary</td>
<td colspan="3">Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed</td>
</tr>
</table>

Multiplication- Year 6

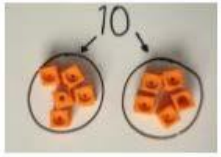
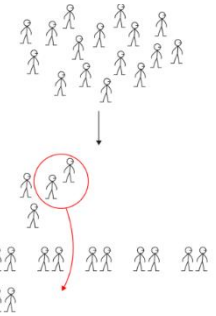
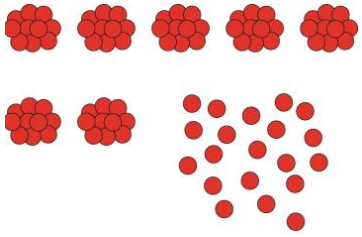
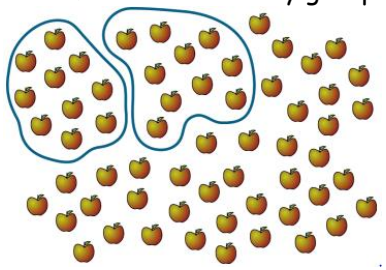
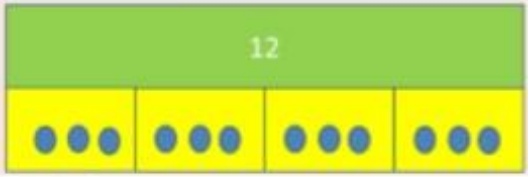
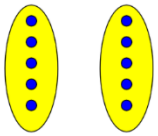
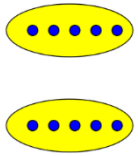
Objective and Strategy	Concrete	Pictorial	Abstract
Multiply decimal up to 2 decimal place by a single digit.	 Create the calculation using ones then make the calculation 10 times smaller by replacing the ones with tenths.  3.3 × 3 =  Use the grid method to layout the calculation. Then add together, making any exchanges needed. 3 x 1.212  	 The calculation will be shown alongside the model chosen to see the connection. **Model** | **Calculation**	Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer.   3 multiplied by 3 hundredths = 9 hundredth 3 multiplied by 8 tenths = 24 tenths 20 tenths can be exchanged for 2 ones 3 multiplied by 2 ones = 6 ones
vocabulary	Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed		

Division

Division- EYFS

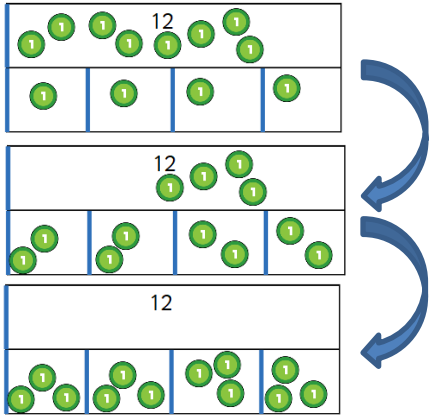
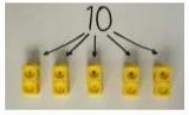
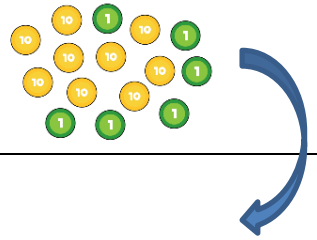
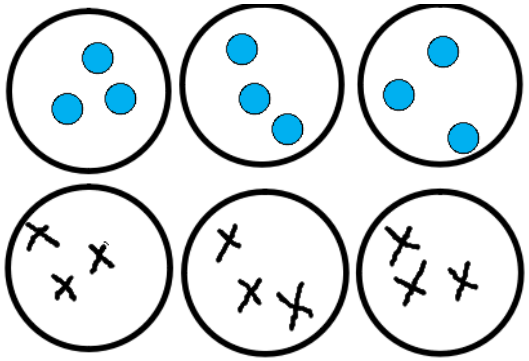
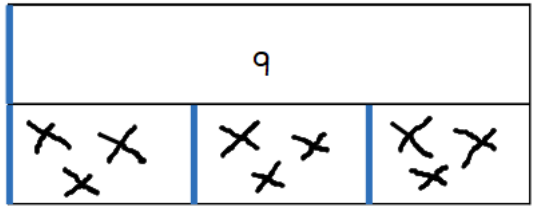
Objectives	Concrete	Pictorial	Abstract
ELG: Numerical Patterns Children at the expected level of development will: Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.	   Children have the opportunity to physically cut objects, food or shapes in half.  Counting and other maths resources for children to share into two equal groups.  Use visual supports such as halving mats and part part whole, with the physical objects and resources that can be manipulated.   Counting and other maths resources for children to explore sharing between 3 or more.	 Pictures and icons that encourage children to see concept of halving in relation to subitising, addition and subtraction knowledge. i.e. Knowing 4 is made of 2 groups of 2, so half of 4 is 2.  Bar model with pictures or icons to support understanding of finding 2 equal parts of a number, to further understand how two halves make a whole.  Pictures for children to create and visualise 3 or more equal groups.	

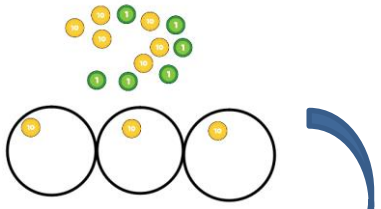
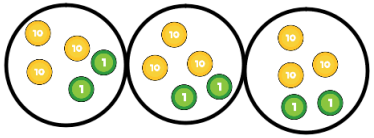
Division- Year 1

Objective and Strategy	Concrete	Pictorial	Abstract
Division as sharing (sharing objects into groups)	   I have 10 cubes, can you share them equally in 2 groups? Use the children to explore how many groups of 2 we can create in the class (no remainders) Check the total value by counting backwards checking that 0 has been reached once all objects have been counted off. Move items into the counted area.  Use counting in tens to count objects efficiently, move objects into groups (no remainders!). Check the total value by counting backwards checking that 0 has been 	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$ Children can use pictorial and concrete resources and circle groups of 2, 10 , 5 3 etc. Then find out how many groups.  Children use bar modelling to show and support understanding.  $12 \div 4 = 3$	**Share 9 buns between three people.** $9 \div 3 = 3$ Use concrete and or pictorial resources to support the abstract. Children record division jottings in number array format. $10 \div 2 =$  **OR**  Children start by drawing dots to represent the first number and then group them.

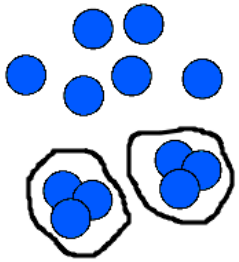
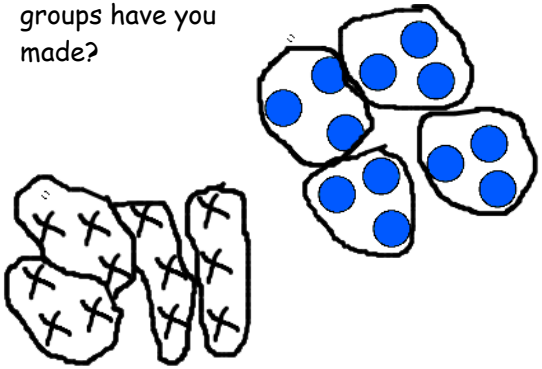
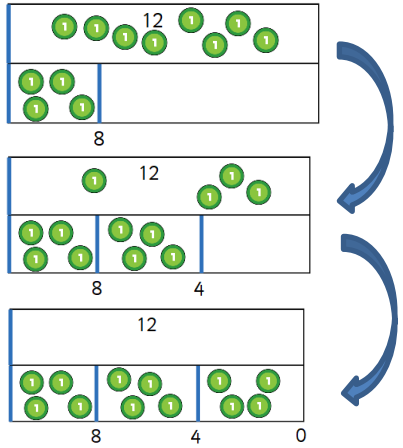
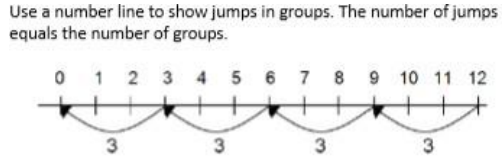
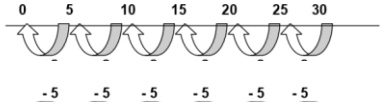
	reached once all objects have been counted off. Move items into the counted area. See NCTEM teaching spines for more information.		
Vocabulary	share, share equally, one each, two each…, group, groups of, lots of, array, divide, equal groups, arrays		

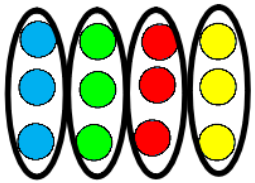
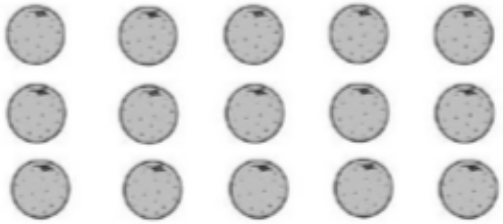
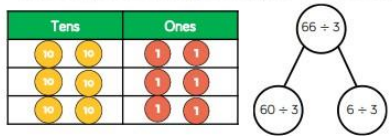
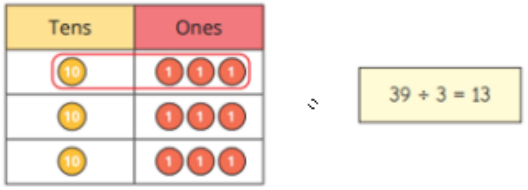
Division- Year 2

Objective and Strategy	Concrete	Pictorial	Abstract
It is important that the children see dividing as both sharing and grouping. Division as sharing	Collect the total number of counters and share them out onto the bar model, in this context the divisor represents the number of groups. Placing one into each section until the total has all gone. 12 ÷ 4 = 3  Divide quantities into equal groups by sharing out the total into the number of groups needed one item at a time.  96 ÷ 3 	Children draw the dots or crosses onto the bar model or into sorting circles and adding one per section until the total has been shared out. How many are in each group that you have made?   	**28 ÷ 7 = 4** **Divide 28 into 7 groups. How many are in each group?** Use concrete or pictorial to support calculations until multiplication is secure.

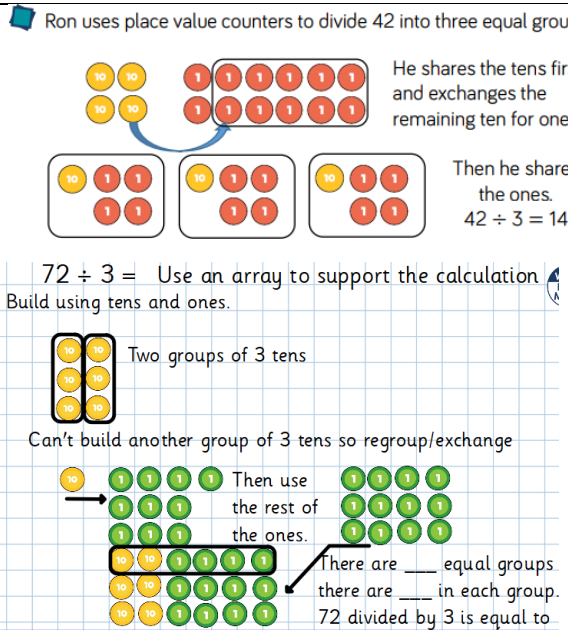
	 Share out tens first one at a time then ones.  A similar method could be used on a number line laying out counters one at a time.  		
Vocabulary	share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, sharing, equal groups of, number line, left, left over, remainder		

Division- Year 3

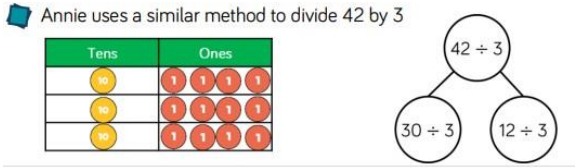
Objective and Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 Lay out the whole using Numicon then using the Numicon piece to represent the divisor lay these on top or below until there is none of the whole left. How many equal groups have you made? Start with the whole (total) then form groups of the divisor until there is nothing remaining/left. Move the groups to a different place as you put them into equal groups. How many groups do you have? 	Draw the whole and then draw round equal groups to represent the divisor. How many groups have you made?  Create a bar model, then subtract an equal sized group at a time record how many you have left each time. Keep going until you have none left. 	Use repeated subtraction to remove equal groups of the divisor from the whole. Represent on the number line.  Children begin to construct a blank number line in order to use a repeated subtraction method to solve simple division. Children are taught that you subtract chunks or groups in order to divide. e.g. 30 ÷ 5 =  You then count up how many chunks of 5 you have subtracted to find the answer. Children should also be taught to use this method when being introduced to the concept of remainders. How many groups of 6 in 24? 24 ÷ 6 = 4

Division with arrays	Use arrays – to lay out the whole building either rows or columns of the divisor.   Link division to multiplication by creating an array and thinking about the number sentenc-es that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4
Divide 2- digit numbers by a 1- digit number by partitioning into tens and ones using a pv Grid.	Eva uses a place value grid and part-whole model to solve 66÷ 3   	See part- whole model – draw out the equivalent of the model used with concrete equipment	

Divide numbers that involve exchanging between the tens and ones. The answers do not have remainders.

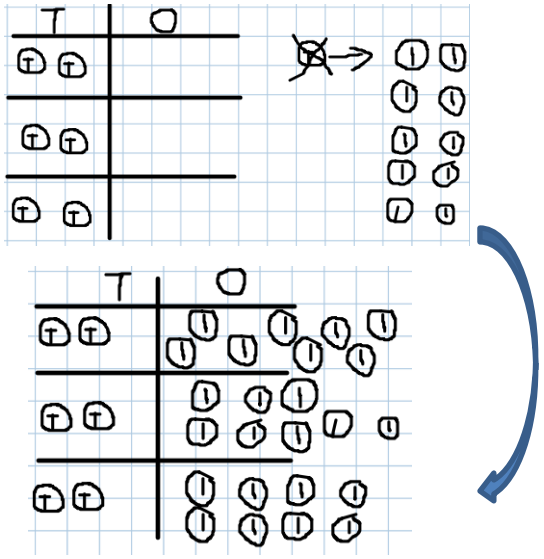


Flexible partitioning. Children use their times-tables to partition the number into multiples of the divisor.



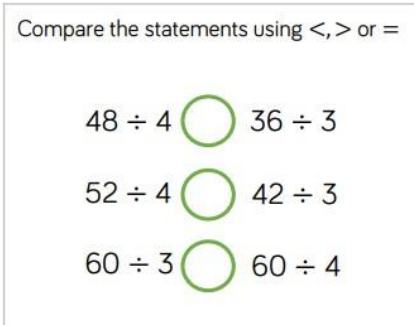
84 ÷ 3 =

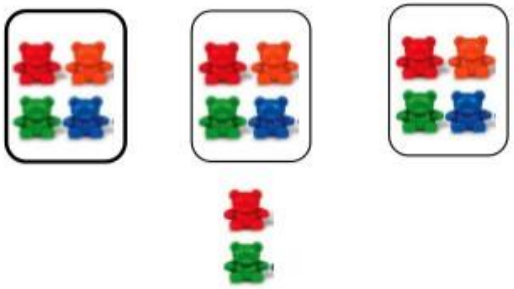
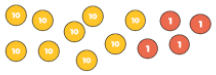
Divide the tens first then exchange the ten that can't be divided, place that into the three groups then share out rest of the ones.



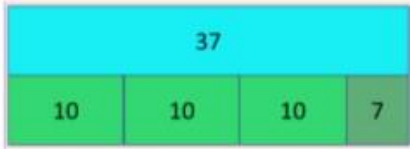
Children may use pictorial representation for the pv counters, alongside the part-whole model
Flexible partitioning can also aid division at this point.

96 ÷ 8
96 ÷ 4
96 ÷ 3
96 ÷ 6



<table>
<tr>
<td>Division with a remainder</td>
<td>
14 ÷ 3 =
Divide objects between groups and see how much is left over

Moving on to:
Use place value counters to work out 94 ÷ 4
Did you need to exchange any tens for ones?
Is there a remainder?

<table>
<tr><th>Tens</th><th>Ones</th></tr>
<tr><td></td><td></td></tr>
<tr><td></td><td></td></tr>
<tr><td></td><td></td></tr>
<tr><td></td><td></td></tr>
</table>
29
</td>
<td>
Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.

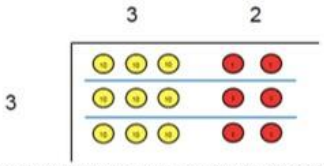
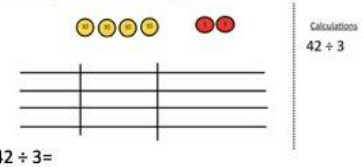
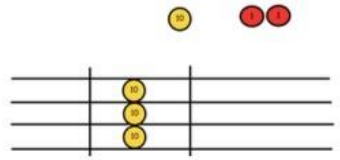
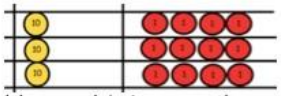
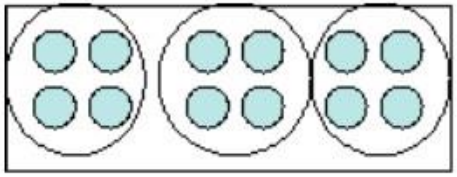
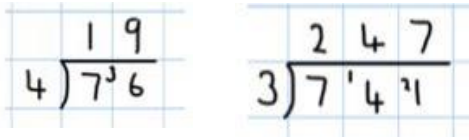
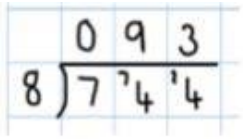
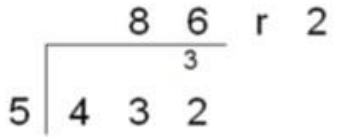

Draw dots and group them to divide an amount and clearly show a remainder.


Use bar models to show division with remainders.


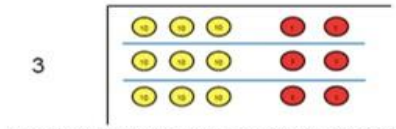
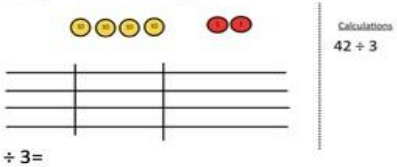
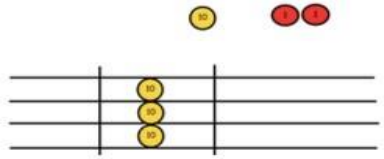
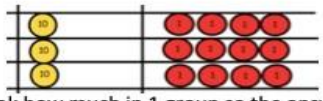
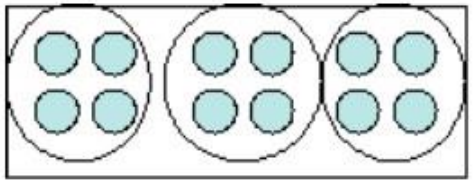
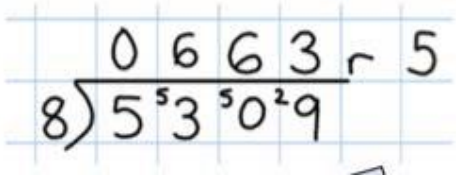
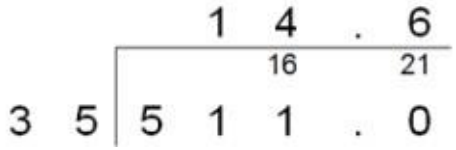
</td>
<td>
Complete written divisions and show the remainder using r.


</td>
</tr>
<tr>
<td>Vocabulary</td>
<td colspan="3">share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product</td>
</tr>
</table>

Division- Year 4

Objective and Strategy	Concrete	Pictorial	Abstract
Divide up to 3 digit numbers by 1 digit. Short Division	96 ÷ 3 Tens Units 3 2 3  Use place value counters to divide using the bus stop method alongside  Calculations 42 ÷ 3 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder  Children should be aware that a 0 is used to keep place value, if the number is not divisible.  Move onto divisions with a remainder. 
Vocabulary	share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive		

Division- Year 5

Objective and Strategy	Concrete	Pictorial	Abstract
Divide at least 4 digit numbers by 1 digit. Interpret remainders appropriately for the context Short Division	96 ÷ 3 Tens Units 3 2 3  Use place value counters to divide using the bus stop method alongside  Calculations 42 ÷ 3 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	 Finally move into decimal places to divide the total accurately. 
Vocabulary	share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method.		

Division-Year 6

Objective and Strategy	Abstract
Long Division	Step 1 – a remainder in the ones

```
   h t o
   0 4 1 R1
4 ) 1 6 5
```

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

```
    th h t o
    0 4 0 0 R7
8 ) 3 2 0 7
```

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

```
     h t o
     0 6 1
4 ) 2 4 7
    - 4
      3
```

When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3.

Check: 4 × 61 + 3 = 247

```
     th h t o
     0 4 0 2
4 ) 1 6 0 9
    - 8
      1
```

When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

Step 2 – a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2)5 8	t o 2 2)5 8 - 4 1	t o 2 9 2)5 8 - 4 ↓ 1 8
Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2)5 8 - 4 1 8	t o 2 9 2)5 8 - 4 1 8 - 1 8 0	t o 2 9 2)5 8 - 4 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

Step 3 – a remainder in any of the place values

	<table><tr><th>1. Divide.</th><th>2. Multiply & subtract.</th><th>3. Drop down the next digit.</th></tr><tr><td>h t o 1 2)2 7 8 Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.</td><td>h t o 1 2)2 7 8 - 2 0 Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.</td><td>h t o 1 8 2)2 7 8 - 2 ↓ 0 7 Next, drop down the 7 of the tens next to the zero.</td></tr><tr><th>Divide.</th><th>Multiply & subtract.</th><th>Drop down the next digit.</th></tr><tr><td>h t o 1 3 2)2 7 8 - 2 0 7 Divide 2 into 7. Place 3 into the quotient.</td><td>h t o 1 3 2)2 7 8 - 2 0 7 - 6 1 Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.</td><td>h t o 1 3 2)2 7 8 - 2 0 7 - 6 1 8 Next, drop down the 8 of the ones next to the 1 leftover ten.</td></tr><tr><th>1. Divide.</th><th>2. Multiply & subtract.</th><th>3. Drop down the next digit.</th></tr><tr><td>h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8 Divide 2 into 18. Place 9 into the quotient.</td><td>h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8 - 1 8 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.</td><td>h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8 - 1 8 0 There are no more digits to drop down. The quotient is 139.</td></tr></table>
Vocabulary	share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method.

Minimal Resources required to support the CPA approach (depending on year group):

- 10 frames (including egg boxes)
- Straws/pipe cleaners
- Bead strings (to 20 and 100)
- Rekenrek frames
- Base 10/Dienes
- Place value grids
- Double-sided counters
- Part-part whole templates
- Place value counters (KS2)
- Multi-link cubes
- Cuisenaire

Useful links to websites which show how to use equipment in different contexts:
https://www.mathswithmum.com/cuisenaire-rods/ How to use Cuisenaire rods effectively to support maths calculation and fractions
https://educationblog.oup.com/primary/using-manipulatives-to-teach-fractions# Using Numicon to teach fractions including multiplying.